C.1 Simple harmonic motion

Practice Worksheet — name: ________________________ date: _____________

FORMULAS FOR THIS TOPIC

DEFINING EQUATION OF SHM $a = -\omega^2 x$ PERIOD AND FREQUENCY $T = \frac{1}{f} = \frac{2\pi}{\omega}$

MASS–SPRING PERIOD $T = 2\pi\sqrt{\frac{m}{k}}$ SIMPLE PENDULUM PERIOD $T = 2\pi\sqrt{\frac{l}{g}}$

DISPLACEMENT (HL) $x = x_0 \sin(\omega t + \phi)$ VELOCITY (HL) $v = \omega x_0 \cos(\omega t + \phi)$

VELOCITY–DISPLACEMENT (HL) $v = \pm\omega\sqrt{x_0^2 - x^2}$ TOTAL ENERGY (HL) $E_T = \frac{1}{2}m\omega^2 x_0^2$

POTENTIAL ENERGY (HL) $E_p = \frac{1}{2}m\omega^2 x^2$

SECTION A — MULTIPLE CHOICE

A1. A particle performs SHM. Its acceleration is greatest when:

- (A) Its speed is greatest
- (B) It passes through the equilibrium position
- (C) Its displacement is greatest
- (D) Its kinetic energy is greatest

A2. A pendulum clock is taken from Earth to the Moon ($g_{Moon} \approx g/6$). Its period will:

- (A) Decrease by a factor of 6
- (B) Increase by a factor of 6
- (C) Increase by a factor of $\sqrt{6}$
- (D) Stay the same

A3. In SHM, at what displacement is the kinetic energy equal to the potential energy? (x_0 = amplitude)

- (A) $x_0/4$
- (B) $x_0/2$
- (C) $x_0/\sqrt{2}$
- (D) x_0

SECTION B — SHORT ANSWER

B1. Show that a 0.40 kg mass on a spring of constant $k = 250 \text{ N m}^{-1}$ oscillates with a frequency of about 4 Hz. [2 marks]

B2. State the two conditions required for a motion to be simple harmonic. [2 marks]

B3. A particle oscillates with amplitude 5.0 cm and period 2.0 s. Calculate its maximum speed and its speed at $x = 3.0$ cm. [4 marks]

ANSWER KEY

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Section A

A1: Its displacement is greatest — $a = -\omega^2 x$: acceleration is proportional to displacement, so it peaks at the amplitude — exactly where the particle is momentarily at rest. Speed and acceleration peak at opposite ends of the motion.

A2: Increase by a factor of $\sqrt{6}$ — $T = 2\pi\sqrt{l/g}$: reducing g by 6 increases T by $\sqrt{6} \approx 2.4$. The clock runs slow. A mass–spring clock, by contrast, would keep perfect time — its period is independent of g.

A3: $x_0/\sqrt{2}$ — $E_p = \frac{1}{2}m\omega^2 x^2$ equals half the total $\frac{1}{2}m\omega^2 x_0^2$ when $x^2 = x_0^2/2$, i.e. $x = x_0/\sqrt{2} \approx 0.71x_0$ — further out than most students guess.

Section B

B1: $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.40/250} = 2\pi(0.040) = 0.25$ s, so $f = 1/T \approx 4.0$ Hz.

B2: The acceleration (restoring force) must be (1) proportional to the displacement from a fixed equilibrium point and (2) always directed towards that point — summarised by $a = -\omega^2 x$.

B3: $\omega = 2\pi/T = \pi$ rad s^{-1}. Maximum speed $v_{max} = \omega x_0 = \pi \times 0.050 \approx 0.16$ m s^{-1}. At $x = 3.0$ cm: $v = \omega\sqrt{x_0^2 - x^2} = \pi\sqrt{0.050^2 - 0.030^2} = \pi(0.040) \approx 0.13$ m s^{-1}.