A.5 Galilean and special relativity

Practice Worksheet — name: ________________________ date: _____________

FORMULAS FOR THIS TOPIC

LORENTZ FACTOR $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ GALILEAN TRANSFORMATION $x' = x - vt, \quad u' = u - v$

LORENTZ TRANSFORMATION (POSITION) $x' = \gamma(x - vt)$ LORENTZ TRANSFORMATION (TIME) $t' = \gamma\left(t - \frac{vx}{c^2}\right)$

RELATIVISTIC VELOCITY ADDITION $u' = \frac{u-v}{1-\frac{uv}{c^2}}$ TIME DILATION $\Delta t = \gamma \Delta t_0$ LENGTH CONTRACTION $L = \frac{L_0}{\gamma}$

SPACETIME INTERVAL $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$ WORLD LINE ANGLE $\tan\theta = \frac{v}{c}$

SECTION A — MULTIPLE CHOICE

A1. Which quantity is agreed upon by all inertial observers?

- (A) The time interval between two events
- (B) The distance between two events
- (C) The spacetime interval between two events
- (D) The simultaneity of two events

A2. A muon has a proper lifetime of $2.2\ \mu$s and travels at $0.98c$ ($\gamma \approx 5.0$). In the Earth frame its lifetime is approximately:

- (A) $0.44\ \mu$s
- (B) $2.2\ \mu$s
- (C) $11\ \mu$s
- (D) $22\ \mu$s

A3. Spaceship A moves at $0.8c$ relative to Earth, and fires a probe forwards at $0.8c$ relative to itself. The probe's speed relative to Earth is:

- (A) $1.6c$
- (B) c
- (C) $0.98c$
- (D) $0.89c$

SECTION B — SHORT ANSWER

B1. Define proper time interval and proper length. [2 marks]

B2. Muons are created 15 km above the Earth's surface and travel downward at $0.995c$ ($\gamma \approx 10$). Explain, from BOTH the Earth frame and the muon frame, why many muons reach the surface. [4 marks]

B3. On a spacetime diagram, event A occurs at the origin and event B occurs elsewhere. State how the diagram shows whether A could have caused B. [2 marks]

ANSWER KEY

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Section A

A1: The spacetime interval between two events — Time intervals, lengths and simultaneity are all frame-dependent in special relativity. The invariant is the spacetime interval $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$ — every inertial observer computes the same value.

A2: 11 μs — The proper time is measured in the muon's rest frame; Earth observers see it dilated: $\Delta t = \gamma \Delta t_0 = 5.0 \times 2.2 = 11\ \mu\text{s}$. This is exactly why so many muons survive the ~15 km trip to the ground.

A3: $0.98c$ — Relativistic velocity addition: $u = \dfrac{0.8c + 0.8c}{1 + (0.8)(0.8)} = \dfrac{1.6c}{1.64} \approx 0.98c$. Nothing carrying information exceeds c — the formula guarantees it.

Section B

B1: The proper time interval is the time between two events measured in the frame where both events occur at the same position (one clock present at both). The proper length is the length of an object measured in the frame in which the object is at rest.

B2: Earth frame: the muon's lifetime is dilated to $\gamma \Delta t_0 \approx 22\ \mu\text{s}$, long enough to cover ~15 km at almost c. Muon frame: the lifetime is normal, but the atmosphere is length-contracted to $15/\gamma = 1.5$ km, short enough to cross before decaying. Both descriptions predict the same count rate at the ground — a consistency check of the theory.

B3: Draw the light cone through A (world lines of light at 45° when axes are x and ct). If B lies inside or on the cone, a signal at $v \le c$ could connect them and causation is possible; outside the cone the interval is spacelike and no causal link exists.